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Global Gauge Anomalies in two-dimensional Bosonic Sigma Models

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In my talk I gave a quick overview about the article [GSW] written in collaboration with Krzysztof Gawędzki and Rafal Suszek. The first objective of the paper is to define a general framework for gauged sigma models. The target space of such a sigma model is a differentiable stack obtained as a quotient of a smooth manifold M by an action of a Lie group H. The fields are triples (Σ, P, ϕ) consisting of a (closed, oriented) surface Σ, a principal H-bundle $p : P \to \Sigma$ and a smooth, H-equivariant map $\phi : P \to M$. The B-field is an H-equivariant gerbe $\mathcal{G}$ over M with a pseudo-connection. The Feynman amplitudes of the model are defined by the formula

$$\mathcal{A}(\Sigma, P, \phi) := \mathrm{Hol}_\Sigma(p_*(\phi^*\mathcal{G} \otimes \mathcal{I}_A)).$$

Here, A is a connection on P, $\mathcal{I}_A$ is a topologically trivial gerbe over P with connection defined by A, p_* is the pushforward of gerbes provided by the equivariant structure on $\mathcal{G}$, and Hol_Σ denotes the surface holonomy of the pushed gerbe around Σ. Anomalies aries when the amplitudes $\mathcal{A}$ depend on gauge transformations of connection A, i.e. they are not gauge invariant.

The second objective of the paper is to use our formalism in order to detect anomalies and ,,discrete torsion“ in gauged Wess-Zumino-Witten models. The latter arises from different choices of equivariant structures on the same gerbe. In my talk I discussed the case of $SU(2) \times SU(2)$ at level $(k, 2)$ with the adjoint action of $\mathrm{diag}(SU(2))/\mathrm{diag}(SU(2))$, considered by Hori [Hor96]. There, we can explain a sign ambiguity of the partition function found by Hori by detecting two different $SO(3)$-equivariant structures on the relevant gerbe.

References

[GSW] K. Gawędzki, R. R. Suszek and K. Waldorf, *Global Gauge Anomalies in two-dimensional Bosonic Sigma Models*, Commun. Math. Phys., to appear. `[arxiv:1003.4154]`

[Hor96] K. Hori, *Global Aspects of Gauged Wess-Zumino-Witten Models*, Commun. Math. Phys. **182**, 1–32 (1996).